APPROVED
at a meeting of the Scientific Council
NJSC «Al-Farabi KazNU».
Protocol No.__ from ___________.

The program of the entrance exam
for applicants to the PhD
for the group of educational programs
D092 - «Mathematics and Statistics»

I. General provisions

1. The program was drawn up in accordance with the Order of the Minister of Education and Science of the Republic of Kazakhstan dated October 31, 2018 No. 600 "On Approval of the Model Rules for Admission to Education in Educational Organizations Implementing Educational Programs of Higher and Postgraduate Education" (hereinafter referred to as the Model Rules).

2. The entrance exam for doctoral studies consists of writing an essay, passing a test for readiness for doctoral studies (hereinafter referred to as TRDS), an exam in the profile of a group of educational programs and an interview.

Block Points	Баллы
1. Interview	30
2. Essay	20
3. Exam according to the profile of the group of the educational program	50
Total /admission score	100/75

3. The duration of the entrance exam is 3 hours 10 minutes, during which the applicant writes an essay, passes a test for readiness for doctoral studies, and answers an electronic examination. The interview is conducted on the basis of the university before the entrance exam.

II. Procedure for the entrance examination

1. Applicants for doctoral studies in the group of educational programs D092 - Mathematics and Statistics write a problematic / thematic essay. The volume of the essay is at least 250 words.

The purpose of the essay is to determine the level of analytical and creative abilities, expressed in the ability to build one's own argumentation based on theoretical knowledge, social and personal experience.

Types of essays:

- motivational essay revealing the motivation for research activities;
- scientific-analytical essay justifying the relevance and methodology of the planned research;
- problem/thematic essay reflecting various aspects of scientific knowledge in the subject area.

Essay Topics:

1. The Role of Mathematicians on the Way of Formation of Digital Kazakhstan.
2. How do you imagine yourself in the role of a future scientist mathematician?
3. At what course should Kazakhstani mathematical science develop?
4. What will be your contribution as a Doctor of Philosophy (PhD) to the development of the state and science?
5. How do you understand the statement that natural phenomena are written in the language of mathematics?
6. Why are you worthy of an educational grant for the «Mathematics» program?
7. I am a future scientist.
8. What is your vision of science in Kazakhstan in the future?
9. How can mathematics be improved in Kazakhstan?
10. Describe your academic and career plans, as well as any special interests (research, academic, leadership opportunities) that you aspire to pursue as a doctoral student.
11. The personality of the scientist
12. Explain what is meant by the term "scientific ethics". Express your opinion on the principles of scientific ethics.

2. The electronic examination card consists of 3 questions.

List of examination topics

Discipline «Mathematical analysis»

Number sequences. Upper and lower limits. Bolzano-Weierstrass theorem and Cauchy criterion for number sequences. Limit of functions, continuity and uniform continuity of functions. Weierstrass's theorem on a uniform continuous line on a closed interval. Derivative and differential of a function of one variable. The connection between them. Invariance of the form of the first differential. The concept of an inverse function and the formulation of the question. Prove the simplest version of the theorem for the existence of an inverse function. Differentiation of an inverse function of one variable, derivatives of inverse trigonometric functions. A function of many variables. Multiple and repeat limits. The connection between them. Partial derivatives. Differential of a function of several variables. Differentiability of functions of several variables. Differentiation of a complex function of several variables. The concept of an implicit function and the formulation of the question. General theorem on implicit and inverse functions. Jacobian. Changing variables in a multiple integral. Green's formula for double integral. Surface integrals. Basic theorems of integral calculus.

Discipline «Functional analysis»

Metric, linear normed, Banach and Hilbert spaces. Examples of metric, normed, Banach and Hilbert spaces. Sequences and properties of convergent sequences in metric and linear normed spaces. Continuous mappings in metric space. Continuity and compactness in metric spaces. The principle of contraction mappings in metric space. General form of a linear bounded functional in a Hilbert space. Riesz's theorem. Measurable sets and their properties. Measurable functions and their properties. Lebesgue integral. Difference between Lebesgue and Riemann integrals. Spaces $L_p(\Omega)$ and their properties. Linear operators in Banach and Hilbert spaces. Bounded operators, unbounded operators, closed operators. Operator norm.

Discipline «Probability theory and stochastic analysis»

General probability space. Classical and geometric definition of probabilities. Conditional probability. Formula for the product of probabilities. Independent events, independent tests. Total probability formula. Bayes' formula. Random variables. Laws of distribution of random variables. Mathematical expectation of random variables. Dispersion. Repeated independent tests. Bernoulli formulas. General definition of a random process and finite-dimensional distributions of a random process. Wiener process. Finite-dimensional distributions of the Wiener process and the characterization property of the Wiener process. Correlation function of a random process. Properties.

Discipline «Algebra and geometry»

Concepts of algebraic structure. Homomorphisms and isomerphisms of algebraic structures. Group of automorphisms of algebraic structures. Examples. Semigroups. Monoids. Reversible elements. Groups. Cyclic groups. Isomorphisms. Cayley's theorem. Homomorphisms. Kernel and image of homomorphism. Connection with normal subgroups. Related classes. Indexes. Lagrange's theorem and its consequences. Ring. Zero dividers. Comparisons. Residue class ring. Homomorphisms of rings. Field. Field characteristics. End fields. Construction of the Galois field. Relationship. Equivalence relations, properties of equivalence classes. Partial order relation. Linear order. Smallest, largest, minimum and maximum elements. Prove that a finite partially ordered set always has a minimal element. Dirichlet's principle. Inclusion and exclusion formula. Number of elements in the Cartesian product of a finite number of finite sets.

Discipline «Differential equations and equations of mathematical physics»

Existence and uniqueness theorems for solutions to the Cauchy problem for ordinary differential systems of first-order equations. Homogeneous linear ordinary differential equation of the n-th order with variable coefficients. Fundamental system of solutions. Inhomogeneous linear ordinary differential equation of n-th order with constant coefficients. Systems of homogeneous linear ordinary differential equations, properties of solutions. Ostrogradsky-Liouville formula. Statement of boundary value problems for a second order linear ordinary differential equation. Sturm-Liouville problem. Existence and uniqueness theorems for the Sturm-Liouville solution. Existence of eigenvalues of boundary value problems for a linear ordinary differential equation. Definition of the Green's function for the Sturm-Liouville problem and its existence. Solving boundary value problems for an ordinary differential equation using the Green's function. Inhomogeneous systems of linear differential equations. Method of variation of arbitrary constants (Lagrange method). Classification and reduction to canonical form of second order partial differential equations in the case of many variables. Cauchy problem for a parabolic type equation. Fundamental solution of the thermal conductivity operator. Volumetric thermal potential, surface thermal potential and their basic properties. Cauchy problem for an equation of hyperbolic type. The concept of characteristic for an equation of hyperbolic type. Continuation method. Statement and basic methods for solving boundary value problems for an elliptic type equation. Hadamard's example on the ill-posedness of the Cauchy problem for the Laplace equation. Variable separation method. General scheme of the Fourier method. Eigenvalue and eigenfunction problem for the Sturm-Liouville operator. Fourier method for solving mixed problems for equations of parabolic and hyperbolic types. Cylindrical functions. Bessel's equation. Bessel functions. Dirichlet and Neumann problems for the Laplace and Poisson equation. Green's function for the Dirichlet problem, its properties. Solution of the boundary value problem for the Poisson equation using the Green's function. Variation and its properties. Euler's equation. The main lemma of the calculus of variations. The brachistochrone problem. The simplest calculus of variations problem with moving boundaries. Transversality condition. Sufficient conditions for a functional to reach an extremum. Legendre's condition. Variational problems for conditional extremum. The concept of connections. Reduction to the unconditional extremum problem. Lagrange multipliers. Weierstrass's theorem in a Banach space

III. List of used sources

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